Dynamic Real Earnings Management and Investments

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Abstract: Extant analytical work typically studies earnings manipulation in either one-period models or models where manipulation has only short-term effects. However, beyond transitory misreporting of private information, manipulation can also occur through deviations from optimal operations to enhance short-term pay at the expense of long-term value á la "real earnings management." In this paper, we incorporate the long-term adverse consequences of such manipulation on firm value in a dynamic contracting model with capital investments and overturn several established analytical results. Specifically, designing an incentive-compatible contract to prevent manipulation with persistent effects forces investors to contract on capital investments that may be either below or above first-best levels. We predict that overinvestment in working capital is more likely in firms with high cash flow but low Tobin's q. Our findings provide a theoretical explanation for the strong investment-to-cash-flow sensitivity and weak investment-to-q sensitivity observed in empirical studies.

JEL classification: G32, D25, D86, L26

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1 Introduction

Extensive empirical evidence shows that executives often manipulate the financial outcomes they report to investors (e.g., Healy and Wahlen, 1999; Dechow and Skinner, 2000). The analytical literature seeking contractual solutions to this problem typically focuses on either single-period models or models where manipulation has only short-term effects (e.g., Liang, 2000; DeMarzo, Fishman, He, and Wang, 2012; Beyer, Guttman, and Marinovic, 2014; Guttman and Marinovic, 2018; Göx and Michaeli, 2023). However, beyond transitory misreporting of private information, manipulation can also occur through deviations from optimal operations aimed at boosting short-term pay. These actions are described in the empirical literature as "real earnings management" (Roychowdhury, 2006; Cohen, Dey, and Lys, 2008; Zang, 2012) and may have lasting adverse effects on productivity. Prior work considering such persistent frictions (e.g., Williams, 2011, 2015; Marinovic and Varas, 2019) abstracts from the investments in working capital that firms undertake. In practice, negative effects on productivity may impact capital investments and need to be accounted for in optimal contractual solutions. To our knowledge, prior literature has not considered this aspect, and our study aims to fill the gap.

We incorporate the long-term consequences of real earnings management into a dynamic contracting model with capital investments and overturn several established analytical results. In our model, an investor ("she") contracts with an entrepreneur ("he"). The entrepreneur makes contractible capital investments and operates a technology that generates cash flow from the capital. The true productivity of the technology and its evolution over (continuous) time are not observable to the investor, who relies on information from the entrepreneur. At each point in time, the entrepreneur must exert costly effort (e.g., conduct efficiency improvements) to increase productivity. However, without proper incentives, he may exert suboptimal effort to gain private benefits and misreport to conceal his actions. Importantly, the entrepreneur's manipulation has a persistent, long-term negative impact on the business by reducing the growth rate of future productivity.

The contract in our model is incentive compatible, i.e., it prevents suboptimal actions and misreporting. As is common in dynamic models set in continuous time, the incentive-compatibility (IC) conditions ensuring such prevention rely on the entrepreneur's *continuation utility*, which is the expected present value of the stock of all future compensation. However, due to the persistent effects of manipulation, the conditions in our model also depend on an additional state variable: the *stock of future incentives*, representing the expected present value of all future pay-performance sensitivity. Specifically, the investor's incentive-compatible investment policy and the productivity growth rate at any given time depend on the stock of future incentives accumulated up to that point.

The introduction of the persistent effect of agency frictions in our model generates predictions that differ from these in extant dynamic investment models lacking such persistence. First, our model predicts that capital investment can be either lower or higher than its first-best level (i.e., in a world without private information and threat of manipulation), whereas dynamic models without persistence (e.g., DeMarzo, Fishman, He, and Wang, 2012) predict only underinvestment. In particular, our model predicts that overinvestment is more prominent among firms with high cash flow but low Tobin's q, which is consistent with empirical evidence (e.g., Blanchard, Lopez-de Silanes, and Shleifer, 1994). Consequently, our model and its predictions provide a possible explanation for the overinvestment problem in practice from the perspective of agency frictions with persistent effects.

To explain the underlying mechanism for the investment inefficiencies in our model, recall that the entrepreneur's operation effort is determined by the IC conditions that prevent manipulation (i.e., induce the desired effort and truthful reporting). When the stock of future incentives is low, the investor is concerned that the entrepreneur could under-provide effort. Incentivizing higher effort is achieved by contracting on lower investment so that the growth rate—and the associated with it marginal cost of effort—are both low. The opposite happens when the stock of future incentives is large: then, the entrepreneur is tempted to take advantage of the incentives by over-providing effort—this accelerates the productivity growth rate, boosts future cash flow, and ensures high continuation utility. To prevent such deviation, the investor is compelled to implement a growth rate higher than the first-best level, because that increases the marginal cost of additional growth in productivity for the entrepreneur. In other words, overinvestment results from the investor implementing an inefficiently high growth rate to prevent the entrepreneur from exploiting the large stock of future pay-performance sensitivity amassed through the contract.

To our knowledge, the mechanism in this model represents a novel force behind overinvestment. In particular, overinvestment in prior literature is driven by fundamentally different forces. For example, in Kanodia and Lee (1998), overinvestment occurs because excessive observable investments signal higher-than-actual unobservable productivity to investors (see also Bebchuk and Stole, 1993; Kanodia, Singh, and Spero, 2005). In Braun, Göx, Niggeman, and Schäfter (2024), excessive unobservable investments arise when managers are (exogenously) more sensitive to short-term stock prices.[1] These studies are generally static, resulting in equilibria featuring either under- or overinvestment, but not both simultaneously. In contrast, our model is dynamic, and thus the equilibrium path involves time-varying degrees of investment, such that both under- and overinvestment can occur along the same historical path. The dynamic nature of our model also allows us to produce unique implications, such as how investment varies with past histories of cash flows or Tobin's q. Specifically, our equilibrium implies a strong investment-to-cash-flow sensitivity and relatively weak investment-to-q sensitivity, which aligns with empirical observations summarized in Ai, Li, and Li (2017) and Cao, Lorenzoni, and Walentin (2019) but is not accounted for in standard q-theory models, in which investment typically has a strong correlation with Tobin's q and zero correlation with cash flows.

Our paper bridges several strands of research. One of them is the large body of accounting literature studying earnings management. This work typically investigates the market's reaction to earnings manipulation by a manager who cares about market prices for exogenous reasons (e.g., Dye, 1988; Fischer and Verrecchia, 2000; Sankar and Subramanyam, 2001; Kirschenheiter and Melumad, 2002; Ewert and Wagenhofer, 2011; Beyer, Guttman, and Marinovic, 2019). In our model, the objective function of the entrepreneur is endogenously

[1] Related, in the capital budgeting literature, a pre-committed hurdle rate is typically set below the first-best level, leading to underinvestment in the *quality* of the investment project (e.g., Bernardo, Cai, and Luo, 2001; Baldenius, 2003; Baldenius, Dutta, and Reichelstein, 2007; Heinle, Ross, and Saouma, 2014; Bastian-Johnson, Pfeiffer, and Schneider, 2013, 2017). However, the hurdle rate can also be higher than the first-best level, resulting in overinvestment in project quality due to the interaction between adverse selection and moral hazard (e.g., Inderst and Klein, 2007; Dutta and Fan, 2009; Laux and Ray, 2020), from endogenous and dynamic search for investment opportunities (e.g., Feng, Luo, and Michaeli, 2024), or because mitigating it proves prohibitively costly (e.g., Gregor and Michaeli, 2024, 2022). In contrast, our paper focuses on the quantity (size) of the investment.

determined by the contract with the investor. In this respect, our paper relates to prior work incorporating compensation design (e.g., Stein, 1989; Liang, 2000; Dutta and Fan, 2014; Beyer, Guttman, and Marinovic, 2014; Göx and Michaeli, 2023) and debt contracting (e.g., Guttman and Marinovic, 2018). The main focus of this latter body of literature is short-term transitory accounting misreporting. In contrast, we consider real actions undertaken by managers to directly affect profitability despite their long-term negative consequences.

Another strand related to our work is the finance literature on agency-based investment theories, such as DeMarzo, Fishman, He, and Wang (2012), Decamps, Gryglewicz, Morellec, and Villeneuve (2016), Cao, Lorenzoni, and Walentin (2019), Ai, Kiku, Li, and Tong (2021), and others. These studies typically model agency friction as the agent's private control over the drift of the output process, which, unlike our setting, does not have any persistent effect. Consequently, the incentive compatibility condition reduces to a static tradeoff between instantaneous private benefit and continuation utility.

Methodologically, our paper belongs to the literature of continuous-time dynamic agency models with transitory shocks (e.g., DeMarzo and Sannikov, 2006; Biais, Mariotti, Plantin, and Rochet, 2007; Sannikov, 2008; Zhu, 2013) and with persistent but publicly observable shocks to model parameters (e.g., Hoffmann and Pfeil, 2010; Piskorski and Tchistyi, 2010; Rivera, 2020; Feng, 2021). The studies most closely related to our work in terms of methodology are Williams (2011, 2015), DeMarzo and Sannikov (2017), He, Wei, Yu, and Gao (2017), and Marinovic and Varas (2019). These studies assume that the principal observes a noisy signal and that the agent can take private actions with a persistent impact on the future generation of that signal. They utilize a first-order stochastic maximum principle approach involving a change of probability measures—a technique also adopted in this paper.[2] However, these studies mainly focus on the design and implementation of the optimal compensation contract and do not consider investments. In contrast, our paper focuses on the optimal policy regarding capital investments in the presence of persistent private information.

[2] A critical result of this technique is that the agent's IC condition involves at least two state variables: the usual continuation utility and the stock of future incentives. This differs from dynamic agency models with project selection or capital budgeting, such as Varas (2018) and Malenko (2019). In these studies, although the agent's action produces a persistent effect, the incentive for such action can be determined at the time of the action, and a single state variable is sufficient to characterize the optimal contract.

2 Setting

We consider an investor ("she") who contracts with an entrepreneur ("he"). Together, they form a firm. Both parties are risk-neutral and have a discount rate of $r > 0$. Time is continuous. At each point in time t, the investor and the entrepreneur can make a lumpy transfer C_t between one another. If $dC_t > 0$, the investor pays the entrepreneur. If $dC_t < 0$, the entrepreneur distributes dividends to the investor.

Investment. Capital investment I_t is made continuously and is observable. Thus, the investor can contract on I and compensate the entrepreneur for any incurred costs. We assume that the cost of investing, G, incorporates not only the amount of investment made, I, but also an additively separable convex adjustment cost, representing the changes to the firm's existing capital, structure, and operations required due to adjustment in firm size. Formally, we assume $G = I + \frac{\theta}{2K}I^2$, where K represents the firm's existing capital and $\theta > 0$ is a known cost parameter. For more elegant exposition, we restate the cost throughout the analysis as $G = g(i)K$, with $i \equiv I/K$ and $g(i) \equiv i + \frac{\theta}{2}i^2$. This specification uses the *rate* of investment (per unit of firm capital), i_t, rather than the absolute level of investment I_t, as the main control variable.[3]

Accumulation and depreciation of capital. The accumulation of the observable firm capital at time t follows

$$dK_t = (I_t - \delta K_t)dt, \tag{1}$$

where $\delta \geq 0$ is the rate of depreciation. That is, the capital depreciates over time and increases with investing.

Production technology. At time t, the firm generates cash flow Y_t from the available capital through a standard linear technology. In particular, the incremental (gross) cash flow

[3] Such scaling is directly adopted from the existing literature (e.g., Hayashi, 1982, Bolton, Chen, and Wang (2011), DeMarzo et al. (2012) etc.) and is mainly for tractability so that all control variables are linear in firm size. Practically, this reflects the idea that investing implications depend on how large the firm's existing capital is. For example, a 10 million investment in a firm with 100 million in capital ($i = 0.1$) requires less adjustments than the same investment in a smaller firm with only 10 million in capital ($i = 1$).

generated through production is given by

$$dY_t = K_t dA_t, \tag{2}$$

where A_t is the capital productivity.

Capital productivity and manipulation. The investor does not observe the true productivity, A_t, and relies on a report from the entrepreneur, $\widehat{A}_t$.[4] We assume that the reported productivity at time t is given by

$$d\widehat{A}_t = a_t \mu dt + \sigma_t d\widehat{Z}_t, \tag{3}$$

where $\widehat{Z}_t$ is the (implied by the report) path of Brownian motion, μ is the average productivity, and σ_t is the intrinsic (and known) time-varying volatility σ_t. The variable a_t represents an unverifiable productivity-enhancing effort that the entrepreneur would like the agent to exert. The true productivity follows

$$dA_t = (\widehat{a}_t \mu - \rho M_t)dt + \widehat{\sigma}_t dZ_t, \tag{4}$$

where Z_t is the (true) path of Brownian motion. The reported productivity reflects the actual volatility $\widehat{\sigma}_t$ and the actual effort of the agent $\widehat{a}_t$ exerted at a quadratic personal cost $\widehat{a}_t^2 K_t/2$. For technical reasons (illustrated in the Appendix), we assume that the space of effort is compact: i.e., $\widehat{a}_t \in [0, \overline{a}]$, where $\overline{a} > 0$ is a finite but sufficiently large upper bound that is irrelevant in the equilibrium.

The variable M_t in equation (4) represents the "stock of past manipulations" accumulated by time t and the parameter $\rho > 0$ captures the long-term negative effect of manipulations on the future growth of productivity. Let $\Delta\mu_t \equiv a_t\mu - (\widehat{a}_t\mu - \rho M_t)$ represent the discrepancy

[4] In settings such as the one we study, the information reported by entrepreneurs is usually about cash flows and/or accounting earnings rather than productivity per se. However, by equation (2) and observability of K_t, reporting earnings (or cash flow Y_t, since it can be derived from the earnings and the observable depreciation amount δK_t) is tantamount to providing information about capital productivity. That is, although earnings themselves are not direct measures of productivity, they provide a signal that the investor can use to gauge how effectively the firm is using its resources (capital). For example, investors in practice frequently use metrics like Return on Assets (ROA) or Return on Capital Employed (ROCE) to infer productivity from reported earnings.

between the reported and true growth rate of productivity. Then, the stock of manipulation M_t evolves according to

$$dM_t = (\Delta\mu_t - \nu M_t)dt + \left(\widehat{\sigma}_t d\widehat{Z}_t - \sigma_t dZ_t\right), \tag{5}$$

where $\nu \geq 0$ is the dissipation rate of M_t and it holds that $M_0 = 0$ (i.e., there is no stock of manipulation at the onset of the game).

Our specifications of true and reported productivities have two important implications. First, in addition to costly deviation from the desired (by the entrepreneur) level of action, $\widehat{a}_t - a_t$, the entrepreneur in our model can manipulate the volatility by adding $\Delta\sigma \equiv \widehat{\sigma}_t - \sigma_t \geq 0$. We assume that, by doing so, the entrepreneur generates a private benefit $\lambda\Delta\sigma K_t$, where $\lambda \in (0, 1)$ is some known parameter. Second, any manipulation bears *long-term negative consequences on the future growth* of productivity as reflected in the term $-\rho M_t$. The exact roles of these two implications are discussed in detail in Section 3, after deriving the incentive compatibility conditions, and in Section 6, after characterizing the optimal contract.

Exit. The investor can terminate the management role of the entrepreneur at any time T by settling any promised transfers between the two parties up to that point. This will be represented by the variable W_T in the ensuing analysis. After the exit, the size of the firm is fixed, and cash flow is generated according to $d\widetilde{Y}_t = K_T(d\widetilde{A}_t - \rho M_T dt)$ where $d\widetilde{A}_t = \gamma\mu dt + \sigma dZ_t$ with γ capturing the average production efficiency in the public market. A typical example of the exiting strategy is the firm's initial public offering (IPO). With this example in mind, we refer to the time $t < T$ as the "investment" period and to $t \in [T, \tau]$ as the "post-IPO" period below. However, other examples of exit, such as a leveraged buy-out or a SPAC merger, are equally applicable to our setting.

The cash flow during the post-IPO period is split between the investor and the entrepreneur. The entrepreneur is given a fraction $\kappa \in [0, 1]$ of the cash flow to be held for a vesting period τ, which we refer to as the entrepreneur's vesting period. The fraction κ, the period T (the length of the investment period prior to IPO), and τ (the length of the vesting period) are endogenous and specified by the contract.

Exit is a one-time decision that cannot be rescinded. To ensure tractability, we assume that the exit strategy is the only feasible option to end the investment period. This can be interpreted as the cost of bankruptcy being sufficiently high so that the entrepreneur never prefers to voluntarily terminate the relationship with the investor. The purpose of this assumption and the role of the exit strategy will be discussed in detail in Section 6.

Contract and commitment. With respect to the contract between the entrepreneur and the investor, we adopt the following definitions:

Definition 1 (Contract) *A contract specifies:*

- *a pair of stopping times* $\{T, \tau\}$ *for the end of the investment period and the end of the post-IPO period, respectively;*
- *the investor's investment policies* $\{I_t\}_{\{t\in[0,T]\}}$ *as well as her recommended productivity-growth and volatility-control efforts* $\{a_t, \Delta\sigma\}_{\{t\in[0,T]\}}$ *during the investment period;*
- *the transfer policies* $\{C_t\}_{\{t\in[0,T+\tau]\}}$ *during the investment and the post-IPO periods.*

Definition 2 (Incentive Compatible Contract) *A contract is incentive compatible (IC) if it involves no misreporting: i.e.,* $\widehat{a}_t = a_t$, $\Delta\sigma_t = 0$ *and* $\widehat{A}_t = A_t$ *for all* $t \geq 0$.

Following the convention in the dynamic contracting literature, we assume that both parties can fully commit to the contract once it is signed. Limited commitment is discussed in Section 7.

3 Entrepreneur's Problem

We first define the entrepreneur's optimization problem under a given contract. Let $\mathcal{F}_t$ denote the filtration generated by the true evolution of productivity. The entrepreneur's optimization problem then solves:

$$\max_{\widehat{a},\, \Delta\sigma,\, \widehat{A}} \quad \mathrm{E}\left[\int_0^{T+\tau} e^{-rt} \left(u_t dt + dC_t\right) \middle| \mathcal{F}_t \right] , \tag{6}$$

subject to (1), (3) and (4), where

$$u_t = \begin{cases} \left(\lambda\Delta\sigma_t - \frac{\hat{a}_t^2}{2}\right) K_t, & \text{if } t \in [0,T] \\ 0 & \text{if } t > T. \end{cases} \tag{7}$$

This is a challenging problem due to the persistent effect of the entrepreneur's private actions. To tackle it, we adopt the technique developed in Williams (2011) and perform a change of the probability measure from that generated by the true evolution of productivity dA_t to that generated by the reported evolution $d\widehat{A}_t$. Details of this technique are given in the Appendix but the results are summarized as follows: First, the entrepreneur's problem during the investment period can be expressed by two state variables. One is his continuation utility, W_t, defined as:

$$W_t = \mathrm{E}\left[\int_t^T e^{-r(s-t)}(u_s ds + dC_s) + e^{-r(T-t)}W_T \middle| \widehat{\mathcal{F}}_t\right], \tag{8}$$

where, compared to (6), the expectation is taken under $\widehat{\mathcal{F}}_t$, the filtration generated by $d\widehat{A}_t$. The other state variable, denoted P_t, arises from the persistent effect of the entrepreneur's stock of manipulation (M_t) and is given by

$$P_t = \mathrm{E}\left[\int_t^T e^{-(r+\nu)(s-t)}\rho P_s ds + e^{-(r+\nu)(T-t)}P_T \middle| \widehat{\mathcal{F}}_t\right] < 0. \tag{9}$$

Broadly speaking, P_t captures the trade-off between the entrepreneur's current manipulation of the true productivity and its impact on future productivity growth. It is an important variable to keep track of in models with persistent private information.

Proposition 1 *An incentive compatible contract has the following necessary properties: there exist $\mathcal{F}$-adapted processes $\{\phi_t, \beta_t\}$ such that*

$$dW_t = (rW_t - u_t)dt - dC_t + \beta_t\sigma_t K_t dZ_t, \tag{10}$$

$$dP_t = (r+\nu-\rho)\, P_t dt - \phi_t\sigma_t K_t dZ_t, \tag{11}$$

where

$$\beta_t = a_t, \tag{12}$$

$$\phi_t \sigma_t \geq \lambda, \tag{13}$$

$$P_t = -\beta_t K_t. \tag{14}$$

Equations (10) and (11) characterize the laws of motion of the two state variables W_t and P_t, where β_t and ϕ_t capture the sensitivity of each state variable to the reported path of productivity. Equation (12) is the (standard) incentive-compatibility condition for desired level of effort (i.e., $\widehat{a}_t = a_t$): the marginal cost of effort is a_t while the marginal value is the increase in continuation utility by β_t which, as in other dynamic moral hazard models, represents the entrepreneur's pay-performance sensitivity or his "skin-in-the-game." Equation (13) is the IC condition for no risk-shifting. Different from (12), the cost of risk-shifting does not come from the variation of W_t, but from the increase of the stock of manipulation (via its sensitivity ϕ_t), which negatively affects the productivity growth rate in the future.

Equation (14) is the IC condition for truthful reporting of productivity, which highlights the critical difference arising in the presence of persistent private information. In standard problems, a strong pay-performance sensitivity is usually required to prevent the entrepreneur from adopting private actions that are suboptimal for the investor—the more "skin-in-the-game" the entrepreneur has, the less motivated he is to shirk. In contrast, the persistent private information in our model implies that a strong pay-performance sensitivity might not be beneficial because it gives the entrepreneur incentives to boost the cash flow temporarily in order to increase his continuation utility, which has a long-term negative impact on future cash flow growth. Instead, the IC condition restricts the level of pay-performance sensitivity according to P_t. Substituting (14) into (9) yields

$$P_t = \mathrm{E}_t \left[-\int_t^T e^{-(r+\nu)(s-t)} \rho \beta_s K_s ds + e^{-(r+\nu)(T-t)} P_T \right]. \tag{15}$$

That is, P_t can be interpreted as the (negative) expected present value of all future pay-performance sensitivity or, more simply, the *stock of future incentives*. Temporary boosts of cash flow increase the entrepreneur's continuation utility as well as the stock of future pay-performance sensitivity. As a result, low cash flow in the future will result in a more severe punishment. The degree of such punishment is captured by P_t and restricts the level of pay-performance sensitivity the contract can implement at each moment.

4 Investor's Problem

This section characterizes the optimal contract under the first-best (Section 4.1) and the second-best (Section 4.2) derived under the IC conditions from Section 3.

4.1 First-Best Contract

The first-best level of production and investment are achieved when productivity is directly observable by the investor. In this case, the first-best level of effort $a^{FB} = 1$ and the first-best investment i^{FB} coincide with the solution in Hayashi (1982) that solves:

$$g'(i^{FB}) = \max_i \frac{\mu - g(i)}{r + \delta - i}. \tag{16}$$

The assumption of quadratic adjustment cost (i.e., $g(i) = i + \theta i^2/2$) implies

$$i^{FB} = r + \delta - \sqrt{(r+\delta)^2 - \frac{2(\mu - r - \delta)}{\theta}}. \tag{17}$$

The first-best investment is independent of λ, ρ and ν, parameters that are only related to the agency frictions; it is time-invariant and independent of σ_t and the realized history of productivity. The investment period is permanent and the entrepreneur's role is never terminated.

4.2 Second-Best Contract

We now explore the full-fledged case with agency frictions (second-best). We solve by backward induction and begin with the post-IPO period in Section 4.2.1, followed by the investment period in Section 4.2.2.

4.2.1 Post-IPO Period

This period is needed because, unlike W_T, any non-zero residual value of P_T accumulated at the end of the investment period cannot be simply paid out to the entrepreneur at once when the investment period is over. Moreover, the relaxation of P_T must maintain its definition as stock of future incentives β_t, from equation (15). The post-IPO period is thus designed as the minimal setting in which P_T is gradually released over time via a vesting period τ. The main result can be summarized as follows:

Proposition 2 *Given the values of the state variables P_T, K_T, and W_T. The optimal length of the post-IPO vesting period $\tau = -\frac{1}{r+\nu}\ln\left[1+\frac{P_T}{K_T}\left(\frac{r+\nu}{\rho\gamma}\right)\right]$. The investor's payoff at the start of the post-IPO period is given by*

$$F_T = \left(\frac{\gamma\mu}{r}\right)\left[1+\left(\frac{r+\nu}{\rho}\right)\frac{P_T}{K_T}\right]^{\frac{r}{r+\nu}} K_T - W_T \tag{18}$$

for all $P_T \in [-\rho K_T/(r+\nu), 0]$.

The derivation of these results is given in the Appendix but the intuition is as follows. First, because the entrepreneur is risk-neutral, it is optimal to subject him to the maximal degree of exposure to the post-IPO cash flow during the vesting period and release any terminal stock of incentives P_T as rapidly as possible. That is, $\kappa = 1$ during the post-IPO vesting period. The larger (in absolute value) P_T is, the longer the vesting period to release it. In particular, if $P_T = 0$, investment can no longer be sustained due to lack of incentives, and exit must happen immediately. However, since there is no stock of incentives to release, it holds that $\tau = 0$ and the investor receives the perpetuity of the post-IPO cash flow ($\gamma\mu/r$ per unit of capital). Meanwhile, the maximal stock of incentives the entrepreneur can accumulate is $P_T = -\rho/(r+\nu)K_T$. This corresponds to $\tau = \infty$, meaning that the entrepreneur retains permanent full claims to the post-IPO cash flow. For all $-\rho/(r+\nu)K_T < P_T < 0$, the entrepreneur is given a finite vesting period, after which his role is permanently terminated.

Equation (18) provides the boundary condition for any level of stock of incentives at which the investor chooses to exit and illustrates a tradeoff when the investor chooses such exiting point: exiting with a larger stock of incentives (i.e. more negative P_T) prolongs the investment period during which the firm can grow under the management of the entrepreneur. However, a larger stock of incentives at exit also means a longer vesting period in which the entrepreneur has full claims to the post-IPO cash flow. Regardless of where the exit point is, (18) provides the corresponding boundary condition used to derive the solution to the investor's problem during the investment period.

4.2.2 Investment Period

Let F_t denote the investor's valuation during the investment period at time $t \in [0, T]$. Under an incentive-compatible contract, F_t maximizes the expected present value of all the future (net) cash flow, given as:

$$F_t = \max_{\{I_t, a_t, T, \phi_t, \beta_t\}} \mathrm{E}\left[\int_t^T e^{-r(s-t)}(dY_s - dC_s - G_s ds) + e^{-r(T-t)} F_T \middle| \mathcal{F}_t \right], \tag{19}$$

subject to the laws of motion dK_t, dW_t, and dP_t specified in (1), (10), and (11), as well as the IC conditions (12), (13), and (14). The terminal payoff F_T is the initial payoff of the post-IPO period given in (A-20). The investor has quintuple controls: investment I_t, recommended productivity growth effort a_t, stopping time T, sensitivity ϕ_t of P_t and sensitivity β_t of W_t to the reported path of productivity.

In general, F_t is a function of three state variables: P_t, W_t, K_t. However, in our model, these state variables can be separated. First, the assumption of equal discounting means that there is no cost for accumulating W_t. Any promised transfers can be costlessly accrued to and paid out as a lump sum at T, the end of the investment period. That is, $dC_t = 0$ for all $t < T$, and the marginal cost of W is always -1. Second, the adjustment cost, the private benefits/costs to the entrepreneur, and the production technology during the post-IPO period are all assumed to be linear in K. Therefore, the investor's value function can be rewritten as $f(p)K - W$, where

$$p_t \equiv -P_t/K_t \geq 0 \tag{20}$$

represents the "stock of future incentives *per unit of capital*." Given (1) and (11), the stock

p_t evolves according to

$$dp_t = (r + \nu - \rho - i_t + \delta)\, p_t dt + \phi_t \sigma_t dZ_t, \tag{21}$$

and $f(p)$ satisfies a Hamilton-Jacobi-Bellman (HJB) equation implied by (21). The IC conditions (12) and (13) are intact while (14) becomes $\beta_t = p_t$ which, combined with (12), implies that $a_t = p_t$. Finally, the terminal payoff F_T given in (18) can also be written as $F_T = f(p_T)K_T - W_T$ where

$$f(p) = \left(\frac{\gamma\mu}{r}\right)\left[1 - \left(\frac{r+\nu}{\rho}\right)p\right]^{\frac{r}{r+\nu}}. \tag{22}$$

As discussed in Section 4.2.1 above, the investment period ends either when $p_t = 0$ without a vesting period due to the lack of future incentives or when p_t is sufficiently high and the resulting vesting period is sufficiently long. We denote this upper boundary for exit as $\overline{p}$ and note that $0 < \overline{p} \leq \rho/(r+\nu)$, which is the range of the stock of incentives that can be released during the vesting period.[5]

Altogether, the optimal contract during the investment period can be characterized as an ordinary differential equation (ODE) of p_t only, summarized as follows:

Proposition 3 *Under the optimal contract, the investor's value function during the investment period is $F(P,K,W) = f(p)K - W$, where $f(p)$ solves the following HJB equation:*

$$rf(p) = \max_{a,i,\phi}\; a\mu - g(i) + (i-\delta)f(p) + (r+\nu-\rho-i+\delta)pf'(p) + \frac{1}{2}\phi^2\sigma^2 f''(p)\;, \tag{23}$$

with boundary conditions:

$$f(0) = \frac{\gamma\mu}{r} \tag{24}$$

$$f(\overline{p}) = \left(\frac{\gamma\mu}{r}\right)\left[1 - \left(\frac{r+\nu}{\rho}\right)\overline{p}\right]^{\frac{r}{r+\nu}}. \tag{25}$$

[5] The exact choice of $\overline{p}$ depends on the investor's objective and constraints at time zero. For example, suppose the maximal vesting period allowed is some $\overline{\tau} > 0$, and the investor chooses $\overline{p}$ to maximize the initial firm value $f(p_0)$ where $p_0 = \arg\max_p f(p)$, then based on the parameters used in the numerical examples, $\overline{p} = \frac{\rho}{r+\nu}\left[1 - e^{-(r+\nu)\overline{\tau}}\right]$. We illustrate the value functions for the analytically simplest case where $\overline{\tau} = \infty$ in the numerical examples in Section 5.

At optimality, $\phi = \lambda/\sigma$*,* $a = p$*, and the investment policy* $i(p)$ *is given by:*

$$i(p) = \frac{f(p) - pf'(p) - 1}{\theta}\,. \tag{26}$$

The general structure of the optimal contract has several similarities with the one in standard agency-based dynamic investment models. First, the investor's HJB equation is summarized by a second-order ODE with a single state variable. Second, the volatility of the state variable is bounded below by the IC constraint ($\phi_t \sigma_t \geq \lambda$, or equation 13), and the concavity of the value function implies that such constraint is always binding under the optimal contract. Third, the cost of the agency friction manifests in the likelihood of incentive-driven contract termination.[6]

However, the optimal contract in our setting also demonstrates significant differences from the standard models without persistent effects. In particular, the optimal productivity-growth effort a_t is pinned down by the combination of *two* IC constraints: $a_t = \beta_t$ (equation 12) and $\beta_t = p_t$ (equation 14). The latter is a unique result of the persistent effect. Without it, the constraint (12) itself is irrelevant under the optimal contract because the entrepreneur is risk-neutral—the incentive would be costless to the investor, who will always implement the first-best level of effort ($a_t = 1$). However, the fact that manipulation bears a long-term negative effect on the future growth of productivity means that the entrepreneur faces not only a tradeoff between instant utility and compensation but also his future compensation through the stock of future incentives. Since the investment period ends when the stock of future incentives is either too low or too high, it is costly for the investor to link the entrepreneur's future incentives to the reported path of productivity and, thus, to impose any particular level of effort and risk-choice. Finally, unlike the standard models in which the investor's optimal policies are functions of continuation utility W, the investor's optimal policies in our model are functions of the stock of incentives p. The different dynamics and interpretations of two state variables yield very different theoretical predictions, explored in the next section.

[6] For example, in DeMarzo, Fishman, He, and Wang (2012), termination occurs when the entrepreneur's continuation utility is insufficient to incentivize production effort (i.e., $W = 0$). Here, termination also occurs when the stock of future incentives is depleted (i.e., $p = 0$).

5 Model Implications

The optimal contract characterized in the preceding section has empirical implications for capital investments and firm value. This section discusses two sets of such implications: the first pertains to the optimal investment policy compared to its first-best level (Section 5.1), and the second pertains to the model-implied correlation between investment, q-measures, and expected cash flow (Section 5.2). Given that we have derived the optimal incentive-compatible contract in the previous section, we no longer differentiate the investor's controls from the entrepreneur's controls and will refer to all endogenous values (such as $f(p)$) and policies (such as investment rate $i(p)$) as the firm's value and the firm's optimal policies, respectively.

5.1 Firm Value and Optimal Investment

The left panel of Figure 1 plots a numerical example of the value of the firm per unit of capital, $f(p)$. When the stock of future incentives per unit of capital, $p = P/K$, is low, the firm value is low, due to the inefficient contract termination when p reaches the incentive termination boundary $p = 0$. Higher p reduces the risk of inefficient termination. However, when p is too high, firm value starts to decrease due to an inefficiently high level of investment, which is discussed next.

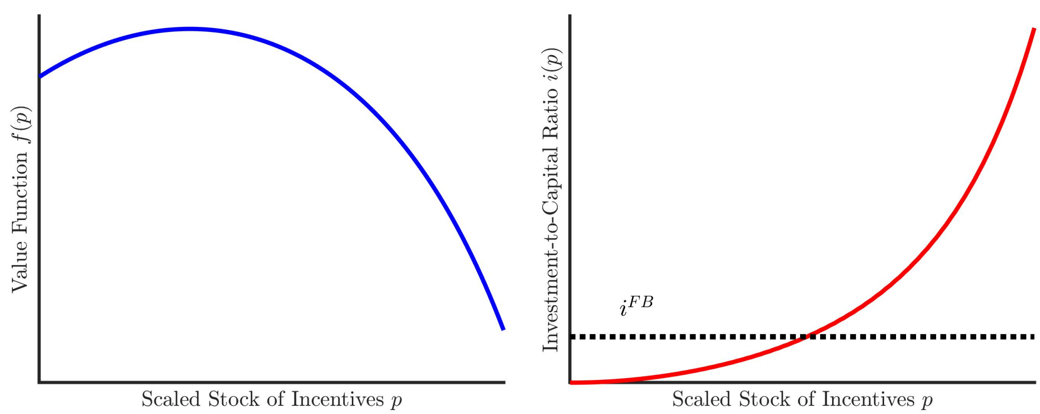

Figure 1: **Value Function and Optimal Investment**

This figure presents the scaled value function $f(p)$ (left panel) and the investment-to-capital ratio $i(p)$ (right panel) under the optimal contract. The parameter values are $r = 5\%$, $\delta = 12.5\%$, $\mu = 20\%$, $\lambda = 0.9$, $\gamma = 0.85$, $\theta = 2$, $\nu = 8\%$, and $\rho = 35\%$.

The right panel of Figure 1 plots the optimal investment policy i as a function of p. For comparison, the first-best level of investment given in equation (17) is also plotted. When the stock of future incentives p is low, the optimal investment intensity is kept low, because the likelihood of the incentive termination is high. Because termination is inefficient, the optimal contract reduces the investment intensity and may even scale down the size of the firm, resulting in $i(p) < 0$. In other words, the optimal contract implies *underinvestment* when p is low.

The optimal investment intensity is a monotonically increasing function of p. This can also been seen from differentiating (26) with respect to p, which yields

$$i'(p) = -\frac{pf''(p)}{\theta} > 0, \tag{27}$$

due to the concavity of $f(p)$. Importantly, $i(p)$ eventually rises above the first-best level, resulting in *overinvestment* from the investor.

Prediction 1 *When the stock of future incentives per unit of capital is sufficiently high, the investor overinvests. Otherwise, she underinvests.*

Our prediction is in sharp contrast to prior dynamic contracting models with capital investment (e.g., DeMarzo, Fishman, He, and Wang, 2012), where agency frictions lead solely to underinvestment.[7] The overinvestment in our model is a unique result arising due to the persistent impact of the agency frictions. That is, accounting for the long-term real effects of earnings manipulation shows that investment distortions can be not only insufficient but might also be excessive.

To illustrate the underlying mechanism for the overinvestment result, recall that the entrepreneur's effort a_t is pinned down by the combination of the IC conditions for desired effort (equation 12) and truthful reporting (equation 14). When the stock of future incentives p is low, the investor prefers a low growth rate because she is primarily concerned with

[7] In DeMarzo, Fishman, He, and Wang (2012), the impact of the entrepreneur's private actions is transitory, and her cost of providing incentives vanishes when her continuation utility is sufficiently high. Because continuation utility only approaches such level from below, investment increases gradually with continuation utility and never surpasses the first-best. This is because the agents in that model are assumed to be more impatient than the principal. Thus, there is a cost for the principal to maintain any level of continuation utility for the agent.

the entrepreneur's under-provision of effort and does not have a sufficient stock of incentives to sustain a high growth rate. As p increases, the exact opposite concern arises: the entrepreneur is tempted to take advantage of the large stock of incentives (pay-performance sensitivity) accumulated by accelerating the productivity growth rate, boosting future cash flow, and consequently receiving higher continuation utility. To prevent such deviation, the investor is compelled to implement a growth rate higher than the first-best level, because that increases the marginal cost of additional growth in productivity for the entrepreneur. In other words, overinvestment results from the investor implementing an inefficiently high growth rate to prevent the entrepreneur from exploiting the large stock of future pay-performance sensitivity amassed through the contract.

In practice, overinvestment is observed in many scenarios, such as aggressive but inefficient mergers and acquisitions (M&As).[8] Blanchard, Lopez-de Silanes, and Shleifer (1994) find that firms with high cash flow may overinvest even when their investment opportunities are poor, as measured by a low Tobin's q. In Section 5.2, we show that high cash flow in our model leads to a higher p but not necessarily a higher q. Consequently, our model and its predictions provide a possible explanation for the overinvestment problem in practice from the perspective of agency frictions with persistent effects.

Table 1 numerically illustrates the key model predictions including the degrees of under- and overinvestment given different parameters. Case I is the baseline setting used to generate Figure 1. In Cases II to IV, all parameters are the same as these in the baseline setting except for the one parameter indicated in each column. Because the deviation from the first-best level of investment stems from agency frictions, we explore the quantitative implications of the model by varying three parameters that only pertain to the agency friction (i.e., parameters that do not affect i^{FB} in equation 17)): λ, which measures the entrepreneur's marginal benefit from risk-shifting; ν, the depreciation rate of the stock of manipulation; and ρ, the marginal impact of manipulation on the growth rate of future productivity.

Table 1 highlights several observations. First, both under- and overinvestment co-exist in all numerical specifications, implying that they are both robust results of the model.

[8] Wang (2018) summarizes the empirical findings on market reactions to M&A news. The reactions appear to be mostly negative for the acquirers, potentially reflecting the concerns of the general investors regarding the value of such expansions.

Cases	I	II	III	IV
Parameters	Baseline	$\lambda = 0.8$	$\nu = 7\%$	$\rho = 40\%$
Model Outputs				
Exit boundary $\overline{p}$	2.69	2.69	2.92	3.08
Optimal initial incentives p_0	1.81	1.95	2.12	2.36
Maximum investor value $f(p_0)$	1.01	1.07	1.10	1.19
Model Predictions				
Maximal degree of underinvestment $i^{FB} - i(0)$	0.40	0.40	0.40	0.40
Maximal degree of overinvestment $i(\overline{p}) - i^{FB}$	3.94	8.42	7.74	9.26

Table 1: This table numerically illustrates the key model outputs and implied optimal investment policies under different parameters. Case I is the baseline setting corresponding to the parameters used in Figure 1. In Cases II to IV, all parameters are the same as these in the baseline setting except for the one parameter indicated in the second row. The optimal initial stock of incentives p_0 is assumed to be the one that maximizes $f(p)$ (i.e., $p_0 \equiv \arg\max_p f(p)$.) The (maximum) degree of underinvestment is defined as the difference between the first-best level of investment (i^{FB} from equation 17, which is constant across all cases illustrated) and the lowest model-implied optimal level of investment (i.e., at $p = 0$). The (maximum) degree of overinvestment is defined as the difference between the highest model-implied optimal level of investment (i.e., at $p = \overline{p}$) and the first-best.

Second, the magnitude of underinvestment is low and insensitive to variations in the degree of the agency friction, while the magnitude of overinvestment is comparably much larger and more sensitive to variations in the agency friction.[9] This suggests that while the benchmark agency models qualitatively explain underinvestment in practice, our model with persistent agency frictions provides a potential mechanism to both qualitatively and quantitatively reconcile overinvestment distortions observed in empirical data. Finally, in each case above, overinvestment is more prominent when the maximal investor value is higher (compared to the baseline case), which is intuitive: accumulating the stock of future incentives is beneficial to the investor only when p is low. When p is sufficiently large, its marginal value becomes negative when the investor faces the possibility of exit, during which time the cash flow must be shared with the entrepreneur. Overinvestment arises to reduce the likelihood of exit and restore firm value to its maximum faster. Therefore, the model predicts that overinvestment should be more prominent among firms with higher potential market value.

[9] The maximal degree of underinvestment, defined as $i^{FB} - i(0)$, is insensitive to parameters that do not affect i^{FB} because $i(0)$ is also insensitive to these parameters including λ, the equilibrium volatility of p_t. See e.g., Figure 3 of DeMarzo, Fishman, He, and Wang (2012). Another potential definition of the degree of underinvestment is the fraction of the state variable under which $i(p) < i^{FB}$, which is more sensitive to the model parameters used in Table 1 but only moderately. Also, this alternative definition makes it difficult to distinguish the sensitivity of underinvestment from that of overinvestment since a large change to the latter mechanically implies a large change of the former.

5.2 Investment's Sensitivity to Tobin's q and Cash Flow

The q-theory is among the most widely adopted theories of firm investment. Two measures of q are commonly studied: the marginal-q (q_m) and the average (Tobin's)-q (q_a), corresponding to the marginal and average value of capital, respectively. In our model, the total value of capital, or the value of the business, is $F(P,K,W)+W=f(p)K$. Thus, both the marginal capital (q_m) and the average Tobin's capital (q_a) can be expressed as simple functions of the state variable p:

$$q_m \equiv \frac{\partial(F+W)}{\partial K} = f(p) - pf'(p), \tag{28}$$

$$q_a \equiv \frac{F+W}{K} = f(p)\ . \tag{29}$$

It is well-known from standard neoclassical models (e.g., Hayashi, 1982) that q_a is identical to q_m in a frictionless environment. However, models with agency frictions predict a wedge between q_a and q_m, implying potential measurement error when using q_a as a proxy for q_m.

When examining investment policies, empirical studies commonly favor the use of Tobin's q_a over the use of marginal q_m mainly because the former is easier to measure. Empirical studies have consistently documented a large sensitivity of investment to cash flow and a small sensitivity of investment to Tobin's q_a. The large investment-to-cash-flow sensitivity is also more prominent among larger and older firms.[10] The above-mentioned empirical findings cannot be reconciled with results arising in prior analytical work with agency frictions (e.g., Bolton, Chen, and Wang, 2011, DeMarzo, Fishman, He, and Wang, 2012) for two reasons. First, these analytical studies assume that the agent's action is binary so that in the equilibrium, the expected cash flow growth rate is exogenous and constant. Second, in these studies, investments and q_a monotonically increase in the state variable. This results in a large investment coefficient on q_a and a zero coefficient on cash flow, which does not align with the empirical findings described above.

[10] Ai, Li, and Li (2017) and Cao, Lorenzoni, and Walentin (2019) summarize the empirical evidence for these observations and offer their explanations. In addition to cash flow shocks, Ai, Li, and Li (2017) introduces a separate productivity shock and a liquidity constraint, while Cao, Lorenzoni, and Walentin (2019) introduces a "news shock" that allows agents to observe the realization of future productivity in advance.

In contrast, the predictions generated by our model are consistent with the empirical observations. To illustrate, the left panel of Figure 2 plots investment i along with q_a and the expected gross cash flow $a\mu$. All three variables are functions of p, but investment and $a\mu$ increase in p, while q_a is hump-shaped in p. This implies a potentially strong correlation between investment and cash flow and a weak or even negative correlation between investment and Tobin's q_a, as formally stated below:

Prediction 2 *There is a strong positive correlation between the optimal investment and cash flow and a weak or even negative correlation between the investment and Tobin's* q_a.

Indeed, after combining (26) and (28) and applying Ito's lemma, the (first-order) variations in investment can be written as the following function of the variations in q_a and expected cash flow:

$$di_t = \eta_1 dq_{a,t} + \eta_2 dE(Y_t), \tag{30}$$

$$\eta_1 = \frac{1}{\theta}, \tag{31}$$

$$\eta_2 = -\frac{1}{\theta\mu}\left[f'(p_t) + p_t f''(p_t)\right]. \tag{32}$$

The investment-to-q_a sensitivity is captured by η_1, which is a constant. The investment-to-cash-flow sensitivity is captured by η_2, which is a function of p and, therefore, depends on the realized cash-flow history. The relative sizes of the two coefficients are illustrated in the right panel of Figure 2. Compared to η_1, η_2 can be very large if $f'(p)$ is very negative or $pf''(p)$ is large in absolute value (recall that $f''(p) < 0$). In particular, both a negative $f'(p)$ and a large (absolute value of) $pf''(p)$ occur when p is high. Thus, the model predicts that larger and older firms, which are often thought to be less financially constrained, will typically have a higher investment-to-cash-flow sensitivity than would smaller and younger firms, as documented in Ai, Li, and Li (2017). Combined with the prediction in Section 5.1 that firms with a larger p also overinvest relative to the first-best, the implications of our model are also consistent with Blanchard, Lopez-de Silanes, and Shleifer (1994), who find that firms with a strong cash flow history may overinvest despite having a low Tobin's q_a.

Table 2 illustrates the model implied investment-to-q_a and investment-to-cash-flow sen-

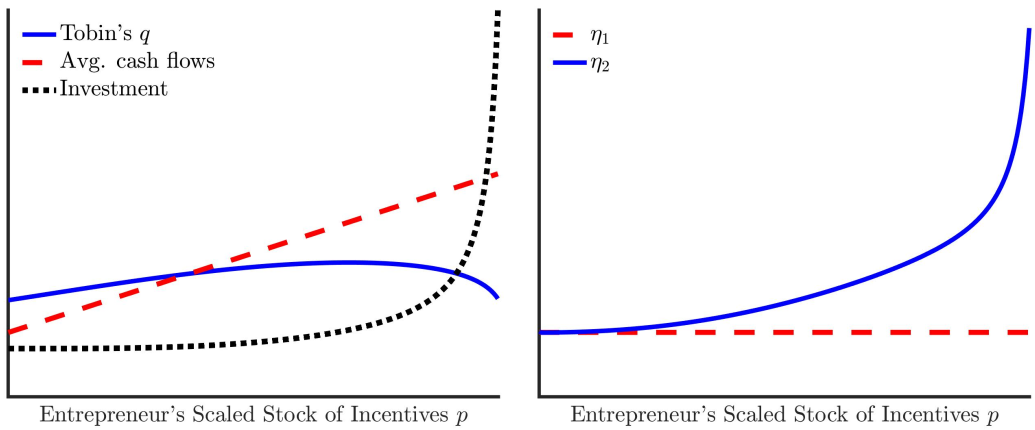

Figure 2: **Investment, Tobin's q, and Cash Flow**
This figure presents the investment i, Tobin's q (q_a), and expected cash flow $a\mu$ in the left panel, and the investment-to-q sensitivity (η_1) and the investment-to-cash-flow sensitivity (η_2) in the right panel. Parameters are the same as these in Figure 1.

sitivities based on numerical simulations. We simulate hypothetical paths of 20 years of history assuming monthly observations starting from $p_0 \equiv \arg\max f(p)$ or until the end of the investment period. Each simulation therefore generates a time-series sample of up to 240 observations based on which the investment sensitivities are calculated. We then repeat each simulation 1,000 times and report the average of key model outputs and predictions in Table 2. As the results show, the model can generate a substantial degree of variations in cash flow but a small degree of variations in Tobin's q_a. Moreover, when an investment becomes more volatile due to a change in the appropriation cost (Cases I and II), the volatility of Tobin's q_a decreases while the volatility of cash flow increases. Combined with the monotonicity of investment and cash flow and the non-monotonicity of Tobin's q_a illustrated in Figure 2, these observations help reconcile the strong investment-to-cash-flow sensitivity and weak investment-to-q_a sensitivity in our model with these documented in the data. Such reconciliation is not possible in models wihout the persistent effect of manipulation (e.g., DeMarzo, Fishman, He, and Wang, 2012). That is, accounting for the real long-term effects of earnings manipulation delivers predictions that are consistent with empirical data.

Cases	I	II	III	IV
Parameters				
Adjustment cost θ	2.00	1.75	2.25	2.25
Expected cash flow μ	20%	20%	20%	21%
Model Outputs				
Volatility of Tobin's q_a Std. q_a	0.20	0.18	0.21	0.22
Volatility of cash flow Std. $a(p)\mu$	0.78	0.82	0.65	0.90
Volatility of investment Std. $i(p)$	0.83	0.86	0.53	0.98
Model Predictions				
Investment-to-$q-a$ sensitivity η_1	0.50	0.57	0.44	0.44
Investment-to-cash-flow sensitivity η_2	1.97	2.21	1.68	2.06

Table 2: This table illustrates the model implied investment-to-q_a and investment-to-cash-flow sensitivities based on numerical simulations. In each case, except for the parameters listed, the values of all other parameters are the same as these in the baseline (Case I) specification in Table 1. The model outputs and predictions represent the average numbers over 1,000 simulated paths, and each simulated path contains up to 240 time-series observations.

6 Discussion of Assumptions

Overall, our model is designed to resemble the agency-based dynamic benchmarks (such as DeMarzo, Fishman, He, and Wang, 2012 and its extensions) as closely as possible to highlight the impact of persistence (i.e., long-term effects of real earnings management) on investment policies. These benchmark models assume risk-neutrality for both the principal and the agent and have the following common properties:

P1. There is a single state variable that characterizes the principal's HJB equation.

P2. The state variable evolves stochastically with non-zero volatility resulting from agency frictions.

P3. There are at least two boundary conditions associated with the principal's HJB equation for low and high values of the state variable, respectively.

The benchmark models achieve P1 with the scaled continuation utility of the agent (i.e., W_t/K_t), by assuming that the production technology and all utility and cost functions are homogeneous of degree one in capital K. Further, they achieve P2 by assuming that the agent controls the drift of the output process (e.g., the cash flow), which links the output volatility to that of W_t. Regarding P3, the lower boundary condition comes from assuming the agent has limited commitment and can quit whenever his continuation utility W_t drops below a certain threshold, which necessitates contract termination at that threshold. The abovementioned studies also assume the agent is more impatient than the principal, thus generating an upper boundary at which cash payments are used in lieu of promised utility to prevent W_t from growing further.

The persistent effect of the agency frictions studied in our paper introduces an additional state variable P_t, which inevitably requires different assumptions in order to maintain the same properties summarized above. To reduce the state space, in addition to the usual homogeneity in K_t, we assume the entrepreneur can both receive compensation and issue dividends to the investor, and the firm is never liquidated voluntarily unless via the exit strategy specified in Section 2. Together, these assumptions allow us to isolate W_t from the investor's value function. As the analysis in Section 4 shows, the model still features incentive-driven termination when the stock of future incentives is insufficient to sustain continuation. The likelihood of this inefficient termination causes optimal investment to deviate from its first-best level and Tobin's q_a to deviate from the marginal q_m—the main implications of limited commitment in the benchmark models without the persistent effect studied in our paper. The volatility of the new state variable is generated by allowing the agent to partially control the volatility of the output (i.e., $\widehat{\sigma}_t$). This links the volatility of P_t to the volatility of W_t when the latter is still linked to the output volatility via the usual drift control (i.e., the effort choice $\widehat{a}_t$). Finally, because P_t represents the stock of incentives instead of compensation, it cannot be simply paid out to the entrepreneur at once as W_t does. Thus, to generate the boundary conditions for the investor's optimization problem, we consider a post-investment period so that P_t can be relaxed gradually through deterministic incentives.

It is worth pointing out that although none of the properties that we introduce is indispensable for the main implications of our model, the comparison with the benchmark models is less transparent if any of the above properties is absent. In other words, it is *possible to design different versions of the model with fewer technical assumptions at the cost of undermining the understanding of the persistent effect.* Meanwhile, existing models of persistent private information (e.g., Williams, 2011, 2015; He, Wei, Yu, and Gao, 2017;

Marinovic and Varas, 2019) all impose various albeit slightly different structures to achieve the same purposes. For example, Williams (2015), He, Wei, Yu, and Gao (2017), and Marinovic and Varas (2019) assume the agent has CARA utility with hidden savings to reduce the dimension of the agent's problem (P1) and to introduce randomness to the state variable (P2).[11] We opt to stay within the risk-neutrality environment as the q-theory benchmarks do, so that none of the different results of our paper can be attributed to the entrepreneur's risk preferences. Similarly, the "post-IPO" technology we use in achieving P3 resembles the "post-retirement" period in Marinovic and Varas (2019) for the exact same purpose, in which the CEO's payment is still tied to the output even though the CEO does not have direct control over the output in that period. Marinovic and Varas (2019) also assume the lengths of the CEO's tenure (T in our model) and retirement (τ in our model) are both exogenous, and the contract is deterministic during the post-retirement period (i.e., β_t is assumed to be known with certainty for all $t \in [T, T+\tau]$).[12] In contrast, we assume the lengths of the investment period and the post-investment vesting period are both endogenous. Despite the differences, the objectives of the post-retirement in Marinovic and Varas (2019) and the post-IPO vesting period in our paper are the same: an exit strategy consistent with the definition of P_t that generates the boundary conditions for the principle's optimization problem during the main stage of the contract.

7 Conclusion

While the success of modern businesses relies heavily on adopting cutting-edge technologies, the rapid development of these technologies, often occurring in various locations around the world, creates an information barrier that prevents investors from obtaining detailed knowledge about the internal production processes of the firms they invest in. This information barrier fosters agency frictions, such as the ability to manipulate financial outcomes through both misreporting and suboptimal private actions with long-term negative consequences—

[11] Under CARA and hidden savings, it is possible to use W_t/P_t as the single state variable. The volatility of W_t, which requires drift control only from the agent, is sufficient to generate randomness in the single-state variable. To our knowledge, no other utility function allows a similar reduction of the state space.

[12] As the authors noted, stochastic relaxation of P_T does not exist so far in the related literature.

i.e., engaging in real earnings management. These frictions can impose not only immediate social costs but also a long-term decrease in the firm's growth.

Our work aims to understand the implications of opaque production technologies on firms' investment policies. Using a dynamic investment model with real earnings management, we demonstrate several implications that are absent in standard models without persistent agency frictions but are consistent with empirical observations. Our model also highlights the critical roles played by certain economic factors typically omitted in standard investment theories, including the investor's ability to control not only the growth rate of the production technology but also its associated risks.

There are several directions in which our model could be extended. The most natural one is perhaps relaxing the investor's commitment requirement. To maintain tractability, the current model assumes that the investor can fully commit to her investment policies, even when under- or overinvestment becomes excessively inefficient and the resulting firm value is low. One potential extension could involve a model in which the investor has the ability to renegotiate the contract in these situations, thus connecting our study to the literature on renegotiation-proofness or limited commitment. We leave such analyses for future research.

Appendix

Proof of Proposition 1: The incentive-compatibility conditions of the entrepreneur's problem can be developed via the stochastic maximum principle technique with a change of measure in Williams (2011). Let $\mathbb{P}$ be the probability measure under the entrepreneur's actions and $\widehat{\mathbb{P}}$ be the probability measure induced by his report, there exists a process η_t such that the Radon-Nikodym derivative between $\widehat{\mathbb{P}}$ and $\mathbb{P}$ is given by

$$\xi_t \equiv \frac{d\widehat{\mathbb{P}}}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_0^t \eta_s^2 ds + \int_0^t \eta_s dZ_s\right) \quad . \tag{A-1}$$

This implies

$$d\xi_t = \eta_t \xi_t d\widehat{Z}_t \tag{A-2}$$
$$dZ_t = -\eta_t dt + d\widehat{Z}_t \quad . \tag{A-3}$$

This technique allows us to evaluate the entrepreneur's expected payoff from any deviation on the probability measure induced by $\widehat{Z}_t$. Consider a contract without intermediate payment (i.e., $dC_t = 0$, because the investor and the entrepreneur share the same discount rate). The entrepreneur's problem during the investment period is therefore

$$\max_{\widehat{a}_t, \Delta\sigma_t, \eta_t} \quad \mathrm{E}^{\widehat{Z}}\left[\int_0^T e^{-\gamma t}\xi_t(u_t dt + dC_t) + e^{-rT}\xi_T W_T\right] \quad , \tag{A-4}$$

subject to (2), (3), (4), (7), and

$$d\xi_t = \eta_t \xi_t d\widehat{Z}_t \tag{A-5}$$
$$dM_t = -\nu M_t dt + \Delta\mu_t dt + \Delta\sigma_t Z_t \tag{A-6}$$
$$= \left(-\nu M_t + (a_t - \widehat{a}_t) + \rho M_t - \Delta\sigma_t \eta_t\right) dt + \Delta\sigma_t \widehat{Z}_t \quad , \tag{A-7}$$

The entrepreneur's optimization problem can be written as the following current value Hamiltonian system:

$$\mathcal{H} = \xi(u + dC) + q^\xi \eta\xi + p^M[(\rho - \nu)M + (a - \widehat{a}) - \Delta\sigma\eta] + q^M \Delta\sigma \quad , \tag{A-8}$$

The adjoint processes satisfies the following Backward Stochastic Differential Equations (BSDE):

$$dp_t^\xi = rp_t^\xi dt - u_t dt - dC_t + q_t^\xi dZ_t \tag{A-9}$$
$$dp_t^M = (r + \nu - \rho)p_t^M dt + q_t^M dZ_t \tag{A-10}$$

with terminal values p_T^ξ, p_T^M. Applying the Feynman-Kac formula to (A-9) implies that

$$p_t^\xi = \mathrm{E}_t\left[\int_t^T e^{-r(s-t)}\left(u_s ds + dC_s\right) + e^{-r(T-t)}p_T^\xi\right] \quad , \tag{A-11}$$

which represents the entrepreneur's continuation utility, and can thus be denoted W_t, following the dynamic contracting literature convention. By the martingale representation theorem, there exists a $\mathcal{F}$-adapted process β_t such that

$$dW_t = (rW_t dt - u_t dt) - dC_t + \beta_t \sigma_t K_t dZ_t \ . \tag{A-12}$$

Similarly, (A-10) implies p_t^M is the solution to

$$p_t^M = \mathrm{E}_t \left[\int_t^T e^{-(r+\nu)(s-t)} \rho p_s^M ds + e^{-(r+\nu)(T-t)} p_T^M \right] \ , \tag{A-13}$$

which represents the discounted marginal value of the persistent impact of misconduct. Define $P_t \equiv p_t^M$ and by the martingale representation theorem, there exists a $\mathcal{F}$-adapted process ϕ_t such that

$$dP_t = (r + \nu - \rho) P_t dt - \phi_t \sigma_t K_t dZ_t \ . \tag{A-14}$$

Finally, taking the first-order derivative of the Hamiltonian system (A-8) with respect to the entrepreneur's controls $\widehat{a}, \Delta\sigma$ and η yields

$$\mathcal{H}_{\widehat{a}} : \quad -aK - p^M = 0 \tag{A-15}$$

$$\mathcal{H}_{\Delta\sigma} : \quad \lambda K + q^M \leq 0 \tag{A-16}$$

$$\mathcal{H}_{\eta} : \quad q^\xi + \sigma p^M = 0 \ , \tag{A-17}$$

Combining (A-15) and (A-17) and using the fact that $q_t^\xi = \beta_t \sigma_t K_t$ yields (12). Substituting q_t^M with $-\phi_t \sigma_t K_t$ implies (13). Finally, substituting q_t^ξ with $\beta_t \sigma_t K_t$ and p_t^M with P_t implies (14).

Proof of Proposition 2: Based on (15),

$$P_T = -\left[\int_0^\tau e^{-(r+\nu)t} \rho \kappa dt \right] K_T = -\frac{\rho \kappa}{r + \nu} \left[1 - e^{-(r+\nu)\tau} \right] K_T. \tag{A-18}$$

The investor's payoff at the start of the post-IPO period is the sum of the expected present value of cash flow during and after the vesting period and the promised transfers to/from the entrepreneur, that is,

$$F_T = \left[\int_0^\tau e^{-rt} (1 - \kappa) \gamma \mu dt \right] K_T + e^{-r\tau} \left(\frac{\gamma \mu}{r} \right) K_T - W_T \tag{A-19}$$

$$= \left(\frac{\gamma \mu}{r} \right) \left[1 - \kappa \left(1 - e^{-r\tau} \right) \right] K_T - W_T. \tag{A-20}$$

For any terminal values (P_T, K_T, W_T), the investor chooses $\kappa \in [0, 1]$ and $\tau \geq 0$ to maximize F_T subject to (A-18). The solution is $\kappa = 1$, and

$$\tau = -\frac{1}{r + \nu} \ln \left[1 + \frac{P_T}{K_T} \left(\frac{r + \nu}{\rho} \right) \right]. \tag{A-21}$$

Substituting these into (A-20) yield the investor's value at the time of exit:

$$F_T = \left(\frac{\gamma\mu}{r}\right)\left[1 + \left(\frac{r+\nu}{\rho}\right)\frac{P_T}{K_T}\right]^{\frac{r}{r+\nu}} K_T - W_T \ , \tag{A-22}$$

for all $P_T \in [-\rho K_T/(r+\nu), 0]$.

Proof of Proposition 3: Given the state variables (P_t, K_t, W_t) and their dynamics (11), (1), and (10), the investor's value function $F(P, K, W)$ solves the following HJB equation:

$$\begin{aligned} rF(P,K,W) = \max_{a,i,\phi,\beta} \ & (a\mu - g(i))\,K + (i-\delta)KF_K - (r+\nu-\rho)PF_P + \frac{1}{2}\phi^2\sigma^2K^2F_{PP} \\ & + (rW - u)F_W + \frac{1}{2}\beta^2\sigma^2K^2F_{WW} + \phi\beta PW\sigma^2K^2F_{PW} \ , \end{aligned} \tag{A-23}$$

subject to IC constraints (12), (13), (14), and boundary conditions:

$$F(0,K,W) = \left(\frac{\gamma\mu}{r}\right)K - W \tag{A-24}$$

$$F(\overline{P},K,W) = \left(\frac{\gamma\mu}{r}\right)\left[1 + \left(\frac{r+\nu}{\rho}\right)\frac{\overline{P}}{K}\right]^{\frac{r}{r+\nu}} K - W \tag{A-25}$$

Conjecture that $F(P,K,W) = f(p)K - W$ where $p = -P/K$. Then $F_K = f(p) - pf'(p)$, $F_P = -f'(p)$, $F_W = -1$, $F_{WW} = F_{PW} = 0$. Substituting these terms into the investor's HJB equation implies $f(p)$ solves:

$$rf(p) = \max_{a,i,\phi} \ a\mu - g(i) + (i-\delta)f(p) + (r+\nu-\rho-i+\delta)pf'(p) + \frac{1}{2}\phi^2\sigma^2 f''(p) \ ,$$

subject to IC constraints (12), (13), and $\beta = p$ (from 14), with boundary conditions:

$$f(0) = \frac{\gamma\mu}{r} \tag{A-26}$$

$$f\left(\overline{p}\right) = \left(\frac{\gamma\mu}{r}\right)\left[1 + \left(\frac{r+\nu}{\rho}\right)\overline{p}\right]^{\frac{r}{r+\nu}} \tag{A-27}$$

$f''(p) < 0$ and the IC constraint (13) implies $\phi = \lambda/\sigma$. The first-order condition for i yields:

$$g'(i(X)) = 1 + \theta i = f(p) - pf'(p) \ , \tag{A-28}$$

which implies (26).

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